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ON THE ROTATION OF GAS AND MAGNETIC FIELDS AT THE SOLAR PHOTOSPHERE

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ABSTRACT

We point out that observations of a 5 percent velocity difference between photospheric gas and magnetic structures at a given latitude may simply result from angular momentum conservation by fluid elements in the convection zone. Estimates of the viscosity and magnetic drag are considered, and we conclude that they probably are not large enough to enforce strictly rigid rotation.
Subject headings: atmospheres, solar — convection — magnetic fields, solar — rotation, solar

I. INTRODUCTION

Measurements of solar rotation from the Doppler shifts of photospheric lines have historically indicated a gas rotation velocity about 5 percent lower than derived from the rotation rate of magnetic active regions at the same latitude. Most recently, the same result has emerged from a series of measurements made over the period 1966–1970 at Mount Wilson (Howard and Harvey 1970; Wilcox and Howard 1970). Since this analysis of the rotation and its time variations was made using a new approach to the measurement of both the gas and magnetic rotation, the persistent difference between the two values has taken on a greater significance. On the basis of these observations, Foukal (1972) suggested that the newly emerged active-region fields might be anchored below the photosphere in a layer of higher angular velocity and could be rotating through the photosphere with low dissipation of the relative motion.

It is the purpose of this Letter to suggest a simple dynamical mechanism that gives rise to the observed effect. Under quite reasonable circumstances, rising and falling elements of fluid in the solar convection zone tend to conserve their angular momentum $m\omega r^2$ as they move up or down. Hence the angular velocity ω varies as $1/r^2$, and one would expect the fluid at the surface to be rotating more slowly than deeper layers. The magnetic structures, being frozen into the matter further down, rotate rigidly with the lower material and hence rotate more rapidly. The observed 5 percent difference would be produced in this picture if the upwelling convecting elements conserved their angular momentum over a zone $\sim 0.05/2R_{\odot} \approx 15{,}000$ km deep beneath the photosphere. It is interesting that this depth is similar to the scale of the supergranulation.

Factors tending to inhibit conservation of angular momentum and enforce rigid rotation are the ordinary viscosity and the dragging effect of the magnetic structures. If the above explanation is correct, these forces must be insufficient to enforce rigid rotation. This problem is considered below.

We note further that the effect is dynamical, being driven by the convective motions, and one does *not* have a characteristic time after which the surface will rotate with the lower layers. As long as there is convection, this effect will occur if the drag forces are small enough.

Observations show essentially the same differential rotation *with latitude* in both the gas and magnetic field. This suggests that the equatorial acceleration of the photosphere (e.g., Howard and Harvey 1970) may be the result of large-scale convective and meridional flows as reviewed by Gilman (1974), whereas the difference between gas and field rotation is produced in a thin surface shell, associated with the supergranular flow observed at the photosphere.

On the other hand, Durney (1974) has shown that if the turbulent viscosity provides an enhanced rate of radial momentum exchange, then large-scale meridional motions can in fact exist which are consistent with the observed values of $\partial\omega/\partial\Theta$, $\partial\omega/\partial r$, and $\partial(\partial\omega/\partial\Theta)/\partial r$ (see also Biermann 1958).

Thus the inward increase of angular velocity that we propose seems to be consistent with present thinking on the mechanisms leading to the solar differential rotation with latitude.

II. ESTIMATE OF VARIOUS DRAG FORCES

We now consider drag effects which tend to force the upper layers to rotate rigidly with the lower layers. We identify two types of drag. First, the slower rotation of the surface layers implies a radial shear and a viscous drag. Second, the rigid rotation of the magnetic field and the consequent interaction between the field structures and the slower-rotating fluid will also tend to drag the fluid along.

a) *Viscous Drag*

Let the viscosity of the fluid be represented by a kinematic viscosity ν. For the moment we will not specify the viscosity, but merely inquire how small ν must be for the viscous drag to be negligible. Since we are interested only in order-of-magnitude estimates, we consider the following simple problem in a *cylindrical* geometry.

Let fluid flow radially between inner radius r_1 and outer radius r_2 with a given velocity v_r, and assume cylindrical symmetry. The inner boundary condition is that the azimuthal velocity v_ϕ is given as $\omega_0 r$, and the outer condition is that the stress $\nu\,(\partial v_\phi/\partial r - v_\phi/r) = 0$. The differential equation for $v_\phi(r)$ is given by Landau and Lifshitz (1959):

$$v_r\left(\frac{\partial v_\phi}{\partial r} + \frac{v_\phi}{r}\right) = \nu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_\phi}{\partial r}\right) - \frac{v_\phi}{r^2}\right). \quad (1)$$

If v_r is a constant, the general solution is

$$v_\phi = \frac{B}{r} + A\,\exp\,(r/r_*)\left(r_* - \frac{r_*^2}{r}\right), \quad (2)$$

where $r_* = \nu/V_r$ is a characteristic length and A and B are to be determined by fitting the boundary conditions. It is readily found that if $r_* \ll r_1$, then

$$v_\phi \approx \omega_0 r_0^2/r \quad (3)$$

to lowest order in r_*/r_1, independent of the sign of V_r. But the v_ϕ given in equation (3) is precisely that representing conservation of angular momentum. Thus, it appears that if ν is small enough to satisfy the inequality

$$\frac{\nu}{v_r r_1} \ll 1\,, \quad (4)$$

then viscous drag is insufficient to maintain rigid rotation.

Putting in typical numbers $r_1 \approx 5 \times 10^{10}$ cm and $v_r \approx 10^4$ cm s^{-1}, one finds that

$$\nu \ll 5 \times 10^{14}\ \mathrm{cm^2\ s^{-1}} \quad (5)$$

will result in conservation of angular momentum, with $v_\phi \propto 1/r$.

Now let us inquire into the magnitude of the viscosity coefficient ν. One may readily compute the molecular viscosity coefficient for the ionized gas from the formula

$$\nu = \frac{2.21 \times 10^{-15} T^{5/2}}{\rho \ln \Lambda}\ \mathrm{cm^2\ s^{-1}} \quad (6)$$

(Spitzer 1962), where ln Λ is the usual Coulomb logarithm. If one takes any reasonable range of values of density and temperature in the convection zone (e.g., Simon and Weiss 1968), one finds that ν is of order unity. Hence molecular viscosity is inadequate by many orders of magnitude to enforce rigid rotation.

It is also necessary to consider eddy, or turbulent, viscosity. The following considerations suggest that this is also too small. First note that the general convective motion cannot contribute to the viscous drag, because each fluid element as it moves up or down tends to conserve angular momentum. In fact, careful consideration of the *molecular* viscosity in this situation shows that the isotropic distribution of velocities is critical. To compute the eddy viscosity coefficient, one must obtain an expression for the rms turbulent velocity v_t relative to the upward- and downward-moving elements, and then find the mean free path λ_t for such motions. The turbulent viscosity ν_t is then given by

$$\nu_t \approx f\lambda_t v_t\,,$$

where f is a factor of order unity. Various values of ν_t are quoted in the literature (see, e.g., Gilman 1974), ranging between 10^{12} and 10^{14} cm^2 s^{-1}. Hence even turbulent viscosity seems insufficient to enforce rigid rotation.

b) *Flux-Tube Drag*

We now consider the drag effect of the magnetic flux tubes moving through the external shell. Both the Reynolds number and the magnetic Reynolds number of the flow are very large (Foukal 1972). Consequently, if there were no convection, the motion of a flux tube of diameter d at relative velocity v through a layer of density ρ would result in a drag force $F_D \approx \rho v^2 d$ per unit depth of the layer (see, e.g., Landau and Lifshitz 1959). If the layer were initially moving at a velocity v relative to the flux tubes, then it would require an impulse of magnitude $\rho v A$ per unit depth (where A is the total area of the layer) to acquire the same velocity as the flux tubes. If this impulse is derived from the force F_D, the layer will require a time of order $t \approx \rho v A/\rho v^2 d$ for its speed to be increased significantly. Now, let the total number of flux tubes be N and let the cross-sectional area of a flux tube be $a = d^2$. Then the total layer will be sped up in a characteristic time $t \approx A/va^{\frac{1}{2}}N$. About 1 percent of the surface is covered by flux tubes and $d \approx 10^3$ km, $v \approx 10^4$ cm s^{-2}, which yield $t \approx 10^6$ s.

Hence, if we think in terms of a rigidly rotating solar core with extruding flux tubes, the spin-up time for an external equatorial shell would be of the order of 10^6 s. Thus, in the absence of convection, an external shell would be forced into corotation with the interior very rapidly.

In the presence of convection, the situation is quite different. The question is simply whether the flux tubes can transfer enough momentum to the gas in one-half convective period to cause it to corotate at the photosphere. Assuming the extreme case, where the convective element came right to the photosphere and experienced the full velocity difference for the full time, we would get once again that the time to get corotation of the gas at the top of one convective cell would be about 10^6 s.

This is long compared with the lifetime of a supergranular convective cell ($\sim 10^5$ s), so we can take the effect of flux-tube drag as being small. However, we also see that a change in the nominal diameter of flux tubes, or in the percentage of the solar surface they cover, from the values discussed above could change this conclusion.

III. DISCUSSION AND CONCLUSIONS

On the basis of the foregoing calculations, it appears reasonable that the observed discrepancy between the rotation velocity of photospheric fluid and magnetic structures can be explained in terms of conservation of angular momentum in radial convection. Estimates of the effect of viscosity and flux-tube drag indicate that they may be insufficient to enforce corotation.

The observed effect can be explained on this basis if the convective elements in the top 15,000 km of the convection zone conserve their angular momentum while the magnetic flux tubes rotate rigidly. The value of this depth suggests that angular momentum is conserved in the supergranular flow. Alternatively, we could have a deeper region where angular momentum is only partially conserved.

In this context, it is of particular interest that in the years 1973–1974, near solar cycle minimum, the photospheric gas velocity seems to gradually approach the rotation velocity of the magnetic field structures (Howard 1974), suggesting that the degree of corotation of the gas and field may vary during the solar cycle.

We suggest that if our simple explanation of the observations is correct, then such *time variations* in the rotation rate of the photosphere (see also Howard 1971) should be related to either variations in the depth of the solar convective zone or variations in the efficiency of angular momentum transfer between the rigid rotator and the convective shell.

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